Recent studies of nuclear effects in deep-inelastic scattering

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Outline

- Brief overview of data on the nuclear EMC effect from charged-lepton DIS.
- Outline a model of nuclear DIS (realistic and yet quantitative). Briefly discuss analysis of data on the EMC effect.
- Application to neutrino physics: theory vs. data comparison for neutrino-nuclear differential DIS cross-sections.

Data on nuclear effects in DIS

- Data on nuclear effects in DIS are available in the form of the ratio $\mathcal{R}(A/B) = \sigma_A(x,Q^2)/\sigma_B(x,Q^2)$ or F_2^A/F_2^B.

- Nuclear targets from ^{2}D to ^{208}Pb

- Experiments:
  – Muon beam at CERN (EMC, BCDMS, NMC) and FNAL (E665).
  – Electron beam at SLAC (E139, E140), HERA (HERMES), JLab (E03103).

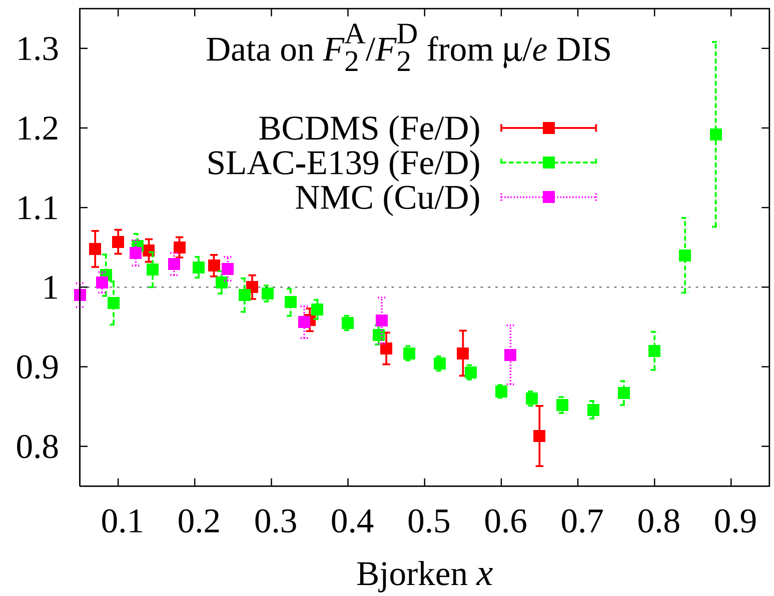
- Kinematics and statistics:
  Data covers the region $10^{-4} < x < 0.9$ and $0 < Q^2 < 150\,\mathrm{GeV}^2$. About 600 data points with $Q^2 > 1\ \mathrm{GeV}^2$ before Jlab E03103 data. E03103 experiment reports new data with about 150 data points for $0.3 < x < 0.9$ and $3 \lesssim Q^2 \lesssim 6\ \mathrm{GeV}^2$.

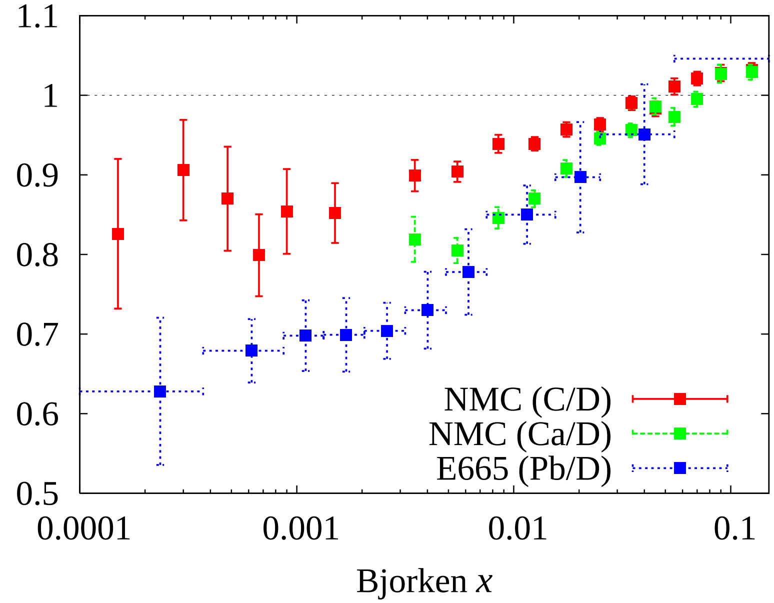
- Additional information on nuclear effects for antiquarks is available from Drell-Yan experiments (E772, E866).

- MINERνA experiment will soon provide us with new and exciting nuclear data from neutrino beam.

Data on the EMC ratios show pronounced A dependence of the ratios $\mathcal{R}(A/D)$ and a weak Q^2 dependence of nuclear effects. Characteristic nuclear effects vs. the Bjorken x:

- Nuclear shadowing at small values of x ($x < 0.05$).
- Antishadowing at $0.1 < x < 0.25$.
- A well with a minimum at $x \sim 0.6 \div 0.75$ (EMC effect).
- Enhancement at $x > 0.75 \div 0.8$ (Fermi motion region).





New data from JLAB E03103 experiment

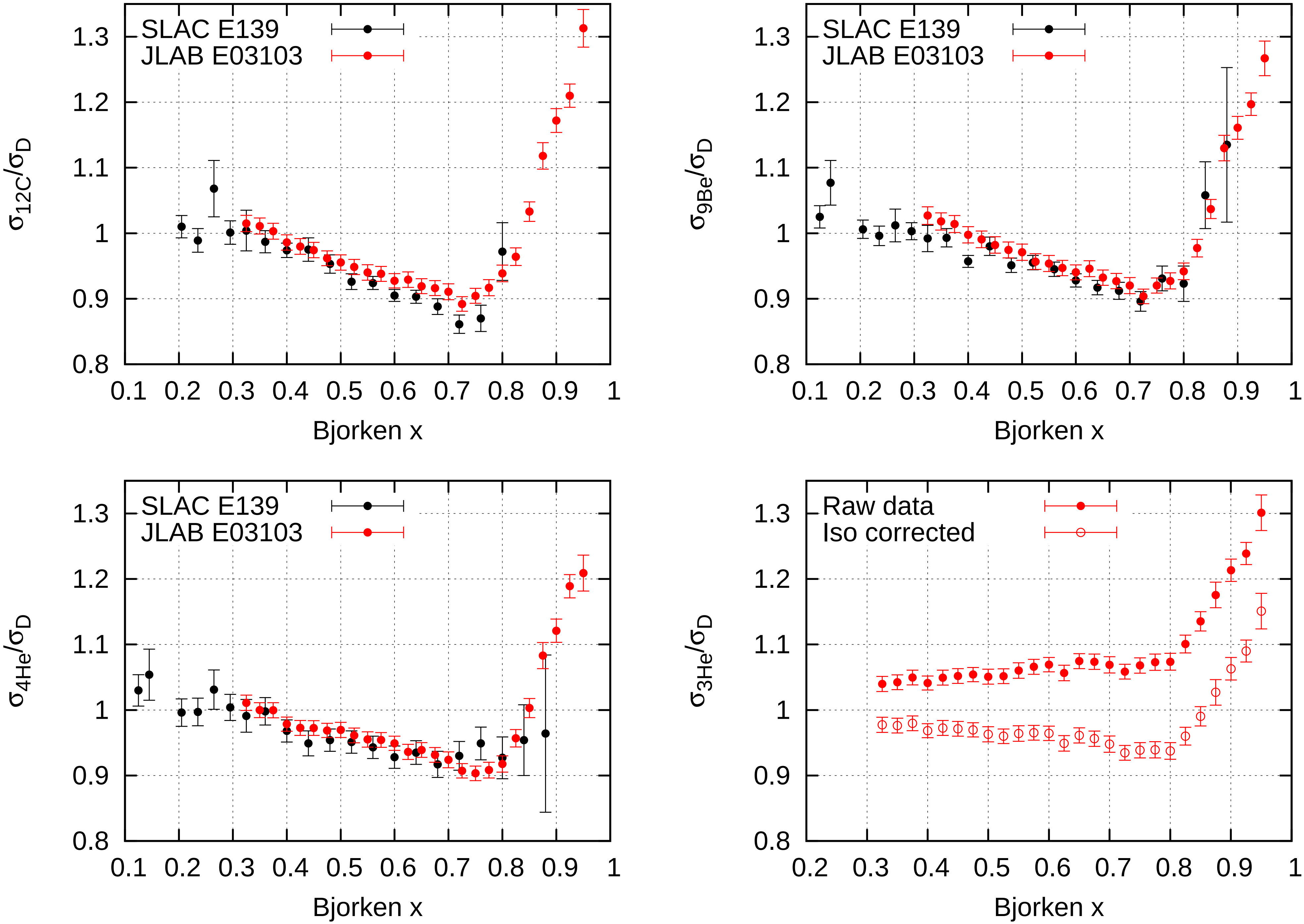
E03103 experiment at Jlab reports the measurement of the EMC ratios for light nuclei: J.Seely, A. Daniel et.al. PRL103,202301,2009

Targets reported: ^{12}C/D, ^{9}Be/D, ^{4}He/D, ^{3}He/D.

Kinematics: Beam energy $E = 5.011$ and 5.766 GeV. Scattering angles are 32, 36, 40, 46, 50 grad.

Overall about 150 data points in the region $0.3 < x < 0.9$ and $2.8 < Q^2 < 7\ \mathrm{GeV}^2$.

Statistics of E03103 experiment at large x is significantly higher than that from previous measurements.

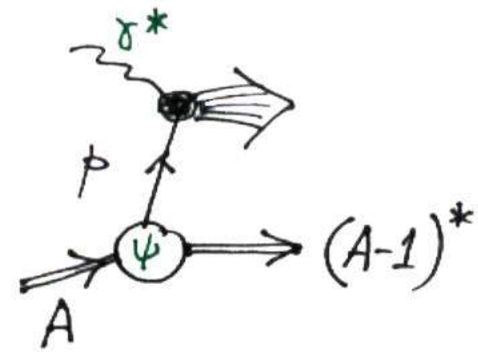

SLAC E139
JLAB E03103
σ_{12C}/σ_D
Bjorken x
SLAC E139
JLAB E03103
σ_{9Be}/σ_D
Bjorken x
SLAC E139
JLAB E03103
σ_{4He}/σ_D
Bjorken x
Raw data
Iso corrected
σ_{3He}/σ_D
Bjorken x

Description of nuclear DIS in Impulse approximation

Fermi motion and nuclear binding corrections (FMB)

$$F_2^A(x,Q^2) = \int \mathrm{d}^4p\, \mathcal{P}_A(p) \left(1+\frac{p_z}{M}\right) F_2^N(x',Q^2,p^2),$$

$$x = \frac{Q^2}{2p\cdot q}, \quad x' = \frac{Q^2}{2p\cdot q} = \frac{x}{1+(\varepsilon+k_z)/M}$$



Similar equations hold in impulse approx. for other structure functions (F_T, F_3). Fermi motion and binding effect is driven by nuclear spectral function

$$\mathcal{P}_A(p) = \sum_n |\psi_n(\boldsymbol{p})|^2 \delta(\varepsilon + E_n(A-1,-\boldsymbol{p}) - E_0(A)).$$

Spectral function describes probability to find a bound nucleon with momentum $\boldsymbol{p}$ and energy $p_0 = M + \varepsilon$.

Nuclear spectral function

Mean-field picture

Nucleus in a first approximation can be viewed as a system of protons and neutrons bound to a self-consistent potential (mean field model, MF). Nucleons occupy the MF energy levels according to Fermi statistics and thus distributed over momentum (Fermi motion) and energy states. MF nuclear spectral function:

$$\mathcal{P}_{\rm MF}(\varepsilon,\boldsymbol{p}) = \sum_{\lambda<\lambda_F} n_\lambda |\phi_\lambda(\boldsymbol{p})|^2 \delta(\varepsilon-\varepsilon_\lambda)$$

where sum is taken over occupied levels with ϕ_λ the wave function and n_λ the occupation number of the level λ (λ_F the Fermi level). MF model is a reasonable approximation if nucleon separation energy and momenta are not high (in nuclear ground state scale, $|\varepsilon| < 50\,$MeV and $p < 300\,$MeV/c).

Fermi gas model:

$$\mathcal{P}_{\rm FG}(\varepsilon,\boldsymbol{p}) = \theta(p_F-|\boldsymbol{p}|)\delta(\varepsilon-V-\boldsymbol{p}^2/2M).$$

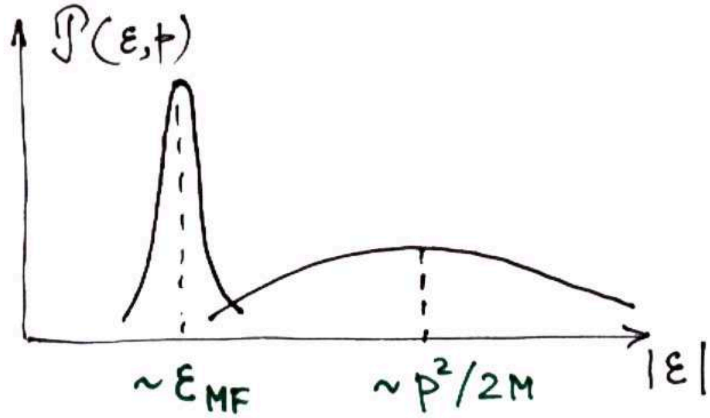
Nuclear spectral function

Nucleon short-range correlation effects

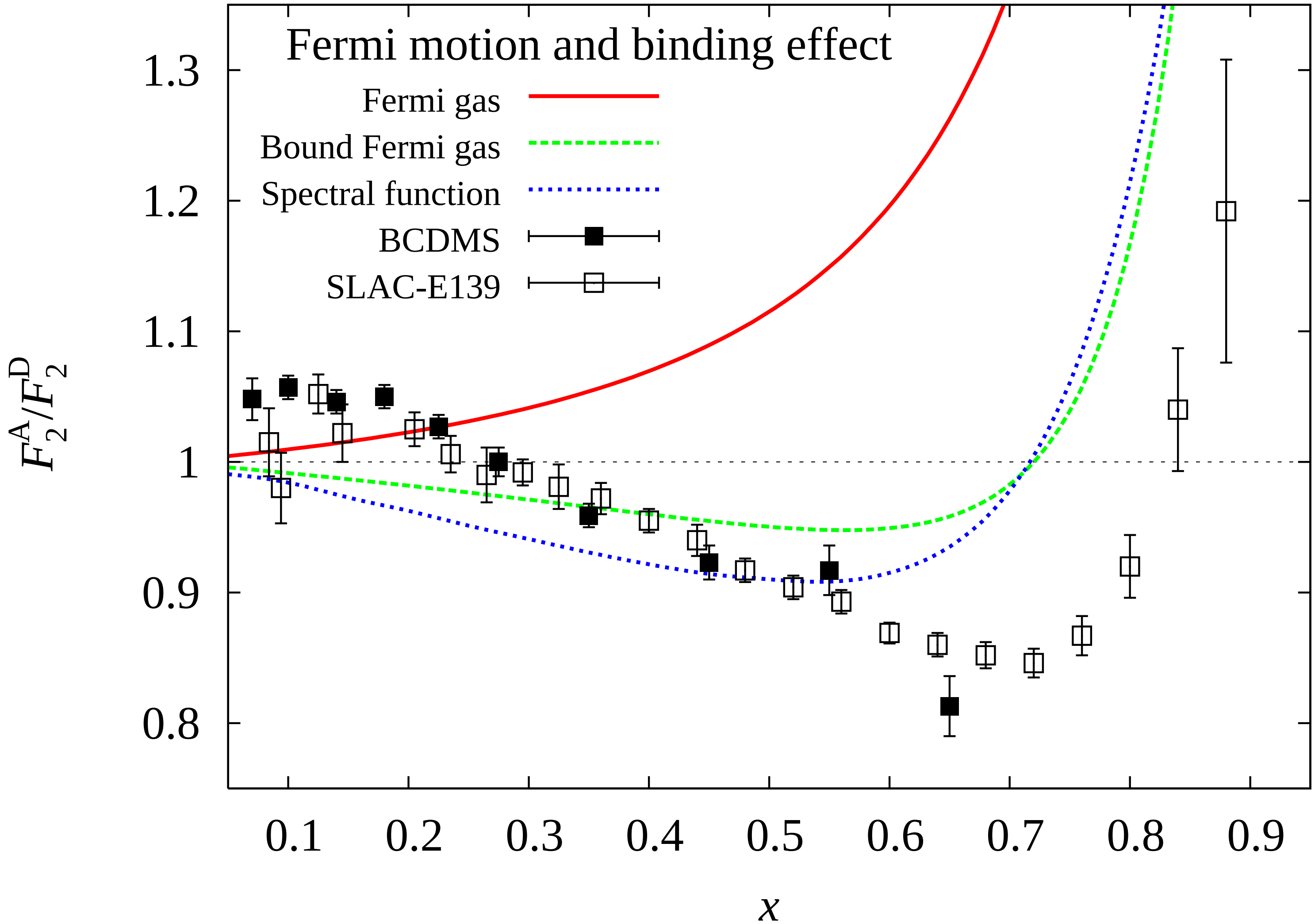
As the separation energy $|\varepsilon|$ becomes higher, the MF approximation becomes less accurate. High-energy and high-momentum component of nuclear spectrum can not be described in the MF model. These effects are driven by short-range NN correlations in nuclear ground state.

$$\mathcal{P}_{\rm cor}(\varepsilon, \boldsymbol{p}) \approx n_{\rm rel}(\boldsymbol{p}) \left\langle \delta\left(\varepsilon + \frac{(\boldsymbol{p}+\boldsymbol{p}_{A-2})^2}{2M} + E_{A-2} - E_A\right)\right\rangle_{A-2}$$

The full spectral function can be approximated by a sum of the MF and correlation parts $\mathcal{P} = \mathcal{P}_{\rm MF} + \mathcal{P}_{\rm cor}$.



EMC Ratio and FMB correction



Although FMB correction gives correct trend, it is not enough for quantitative undestanding of data. IA should be corrected for a number of effects.

Missing light-cone momentum

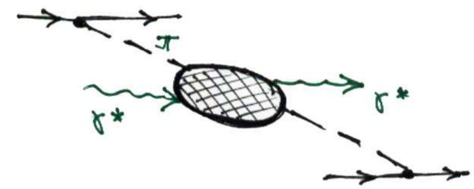
Light-cone momentum of bound nucleon $y = p \cdot q / M q_0 = 1 + (\varepsilon + p_z)/M$. Averaging with the nuclear spectral function gives

$$\langle y \rangle = 1 + \frac{\langle \varepsilon \rangle + \frac{2}{3} \langle \boldsymbol{p}^2 \rangle}{M} \approx 0.95 - 0.96.$$

This indicates the presence of nonnucleon d.o.f. in nuclei which should balance the missing LC momentum. IA is incomplete and should be corrected for this effect.

Nuclear pion effect

Scattering from nuclear meson fields, which mediate interaction between bound nucleons, generate a meson (pion) correction to nuclear structure functions (model calculations in the context of EMC effect by Llewellyn-Smith, Ericsson-Thomas, G.Miller,...).



$$\delta F_i^{\pi/A}(x,Q^2) = \int_x \mathrm{d}y f_{\pi/A}(y) F_i^{\pi}(x/y,Q^2)$$

- Contribution from nuclear pions (mesons) is important to balance nuclear light-cone momentum $\langle y\rangle_\pi + \langle y\rangle_N = 1$.

- The nuclear pion distribution function is confined within a region $y < p_F/M \sim 0.3$. For this reason the pion correction to nuclear (anti)quark distributions is relevant at $x < 0.3$.

- The magnitude of this correction is driven by average number of "pions" $n_\pi = \int \mathrm{d}y\, f_{\pi/A}(y)$. By order of magnitude $n_\pi/A \sim 0.1$ for a heavy nucleus like ^{56}Fe.

- Nuclear pion correction effectively leads to enhancement of nuclear sea quark distribution and does not affect the valence quark distribution (for isoscalar nuclear target).

Nucleon off-shell effect

Bound nucleons are off-mass-shell $p^2 = (M+\varepsilon)^2 - \boldsymbol{p}^2 \neq M^2$. In off-shell region nucleon structure functions and form factors generally depend on additional variable p^2:

A few models were suggested for the off-shell effect in DIS structure functions (Gross–Liuti, Melnitchouk–Schreiber–Thomas, Kulagin–Piller–Weise). We follow a phenomenological approach and constrain the off-shell effect from data assuming the virtuality parameter $v = (p^2 - M^2)/M^2$ to be small (e.g. $\langle v \rangle \sim -0.15$ for ^{56}Fe)

$$F_2^N(x, Q^2, p^2) \approx F_2^N(x, Q^2)\left(1 + \delta f(x)\,\frac{p^2 - M^2}{M^2}\right)$$

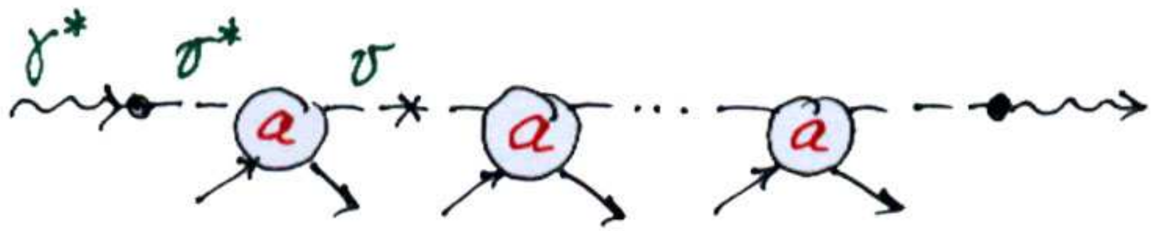
- The off-shell function $\delta f(x)$ makes sense of the response of nucleon parton distributions to variation of the nucleon mass, $\delta f = \partial \ln q(x, p^2)/\partial \ln p^2$.

- Off-shell dependence is closely related to idea of modification of nucleon in nuclear environment. In a simple model $\delta f(x)$ can be directly related to the variation of the nucleon core radius in nuclear environment.

- We extract $\delta f(x)$ from analysis of data on nuclear EMC effect [S.K. & R.Petti, NPA765(2006)126].

Nuclear DIS in coherent regime: shadowing

At small x DIS is driven by $\gamma^* \to v^*$ conversions into virtual hadronic states. Nuclear effects come from multiple interactions of hadronic states during the propagation through matter.



Effect is relevant at small x such as an average time of life (coherence length) of hadronic fluctuation $\tau \sim (Mx)^{-1} \ > $ average internucleon distance $r \sim 1.5\,\mathrm{Fm}$. The onset of the effect is at $x \sim 0.15$, while a developed shadowing effect would require $x \ll 0.1$

The magnitude of coherent effects is driven by effective scattering amplitude a of a virtual hadronic states off the nucleon. Cross-section $\sim \mathrm{Im}\, a$. Behavior in transitional region of $x \sim 0.05 \div 0.1$ is also affected by $\mathrm{Re}\, a$.

The amplitude a is characterized by the helicity state h of the boson ($h = \pm 1$ and $h = 0$ for transverse and longitudinal polarization, respectively). The longitudinal amplitude a_0 determines the structure function F_L, the average $a_T = (a_{+1} + a_{-1})/2$ corresponds to F_1, the asymmetry $a_\Delta = (a_{+1} - a_{-1})/2$ corresponds to F_3.

In addition the amplitude depends on the isospin I (proton and neutron dependence) and C-parity (ν and $\bar\nu$ dependence), $a_h^{(I,C)}$. Note that interaction of virtual photon γ^* is described by a C-even amplitude, and (anti)neutrino interaction involve both C-even and C-odd amplitudes.

Correspondence between $a_h^{(I,C)}$ and the structure functions:

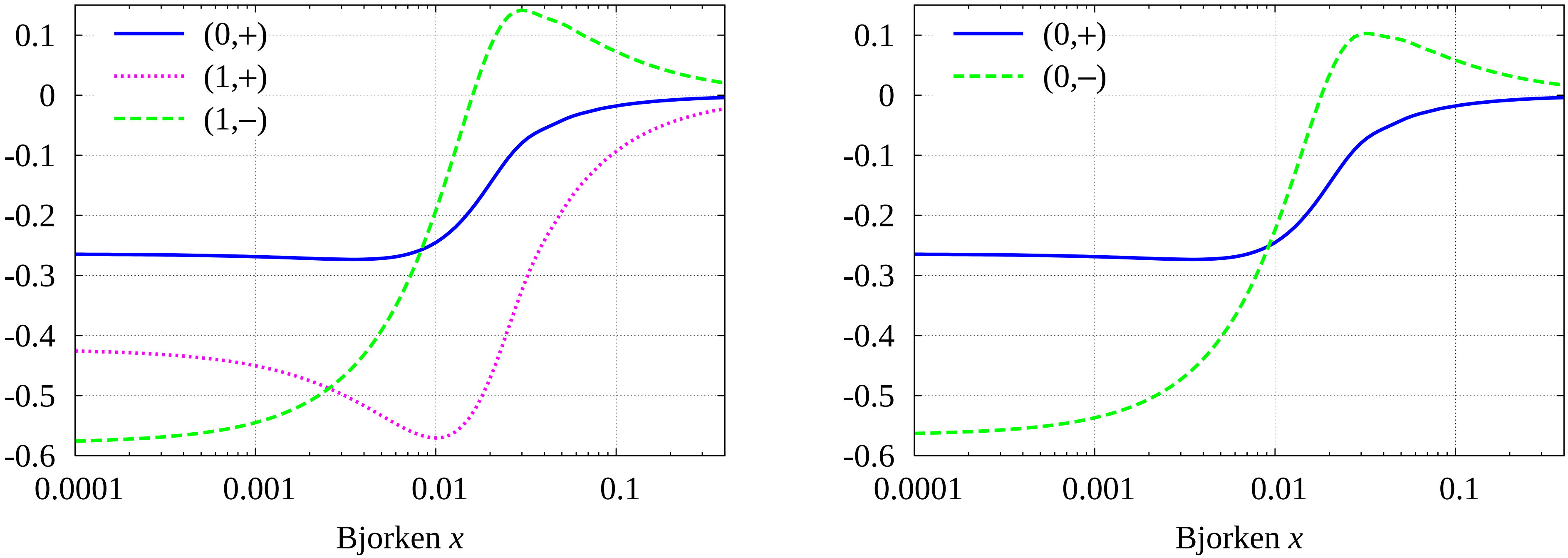
$$a_T^{(0,+)} \longrightarrow F_1^{\mu(p+n)} \text{ and } F_1^{(\nu+\bar\nu)(p+n)}\,, \qquad a_\Delta^{(0,-)} \longrightarrow F_3^{(\nu+\bar\nu)(p+n)}$$

$$a_T^{(1,+)} \longrightarrow F_1^{\mu(p-n)} \text{ and } F_1^{(\nu+\bar\nu)(p-n)}\,, \qquad a_\Delta^{(1,-)} \longrightarrow F_3^{(\nu+\bar\nu)(p-n)}$$

$$a_T^{(0,-)} \longrightarrow F_1^{(\nu-\bar\nu)(p+n)}\,, \qquad a_\Delta^{(0,+)} \longrightarrow F_3^{(\nu-\bar\nu)(p+n)}$$

$$a_T^{(1,-)} \longrightarrow F_1^{(\nu-\bar\nu)(p-n)}\,, \qquad a_\Delta^{(1,+)} \longrightarrow F_3^{(\nu-\bar\nu)(p-n)}$$

Coherent multiple scattering nuclear corrections depend on quantum numbers (C, I).



The relative nuclear correction to transverse effective cross section σ_T calculated for different isospin and C-parity scattering states for ^{208}Pb at $Q^2 = 1\ \mathrm{GeV}^2$. The labels on the curves mark the values of the isospin I and C-parity, (I, C).

Model

Taking into account major nuclear corrections we build a quantitative model for nuclear structure functions (for more detail see S.K. & R.Petti, Nucl.Phys.A765(2006)126)

$$F_i^A = F_i^{p/A} + F_i^{n/A} + \delta_\pi F_i + \delta_{\mathsf{coh}} F_i$$

* $F_i^{p/A}$ and $F_i^{n/A}$ are bound proton and neutron structure functions with Fermi motion, binding and off-shell effects calculated using realistic nuclear spectral function.

* $\delta_\pi F_i^A$ and $\delta_{\mathsf{coh}} F_i^A$ are nuclear pion and shadowing corrections.

In actual calculations we use:

- Free proton and neutron structure functions computed in NNLO pQCD + TMC + HT using phenomenological PDFs and HTs from fits to DIS data (Alekhin).
- Realistic nuclear spectral function which includes the mean-field as well as the correlated part.
- Nuclear pion correction as a convolution of nuclear pion distribution function with pion PDFs.
- Coherent nuclear corrections are calculated using Glauber multiple scattering theory in terms of effective amplitude a_T.

Analysis of EMC effect

⇒ Parameterize unknown off-shell correction function $\delta f(x)$ and effective scattering amplitude a_T responsible for nuclear shadowing. Calculate nuclear structure functions, test with data and extract parameters from data.

⇒ We study the data from e/μ DIS in the form of ratios $\mathcal{R}_2(A/B) = F_2^A/F_2^B$ for a variaty of targets. The data are available for A/D and $A/^{12}$C ratios.

⇒ In our analysis we perform a fit to minimize $\chi^2 = \sum_{\text{data}}(\mathcal{R}_2^{exp} - \mathcal{R}_2^{th})^2/\sigma^2(\mathcal{R}_2^{exp})$ with σ the experimental uncertainty using data with $Q^2 > 1\,\mathrm{GeV}^2$ for the ratios
^{4}He$/D$; ^{7}Li$/D$; ^{9}Be$/D$; ^{12}C$/D$; ^{27}Al$/D$; ^{27}Al$/^{12}$C; ^{40}Ca$/D$; ^{40}Ca$/^{12}$C
^{56}Fe$/D$; ^{63}Cu$/D$; ^{56}Fe$/^{12}$C; ^{108}Ag$/D$; ^{119}Sn$/^{12}$C; ^{197}Au$/D$, ^{207}Pb$/D$; ^{207}Pb$/^{12}$C
overall about 560 points.

⇒ Verify the model by comparing the calculations with data not used in analysis.

Parametrization of off-shell function and effective amplitude:

$$\delta f(x) = C_N(x - x_1)(x - x_0)(x_2 - x)$$

$$a_T = \sigma_T(i + \alpha)/2, \quad \sigma_T = \sigma_1 + \frac{\sigma_0 - \sigma_1}{1 + Q^2/Q_0^2}$$

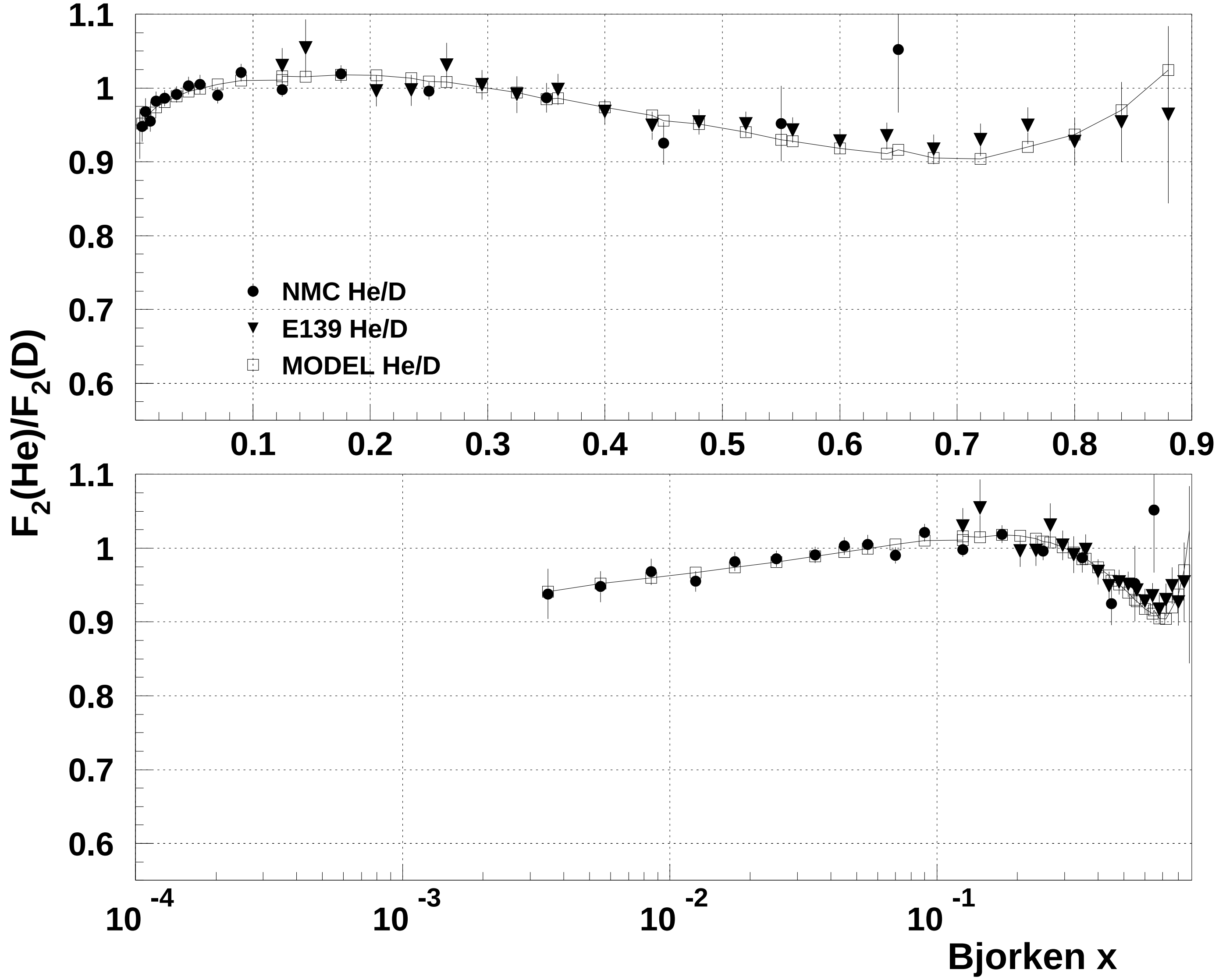
Not all parameters are free or independent.

* We fix $\sigma_0 = 27\,\text{mb}$ and $\alpha = -0.2$ to have the correspondence with VMD model at $Q^2 \to 0$.
* From preliminary trials, the parameter x_2 turned out fully correlated with x_0, $x_2 = 1 + x_0$ fixed in the final fit.
* Best fit gives $\sigma_1 \approx 0$. The correlations between σ_1 and off-shell parameters are negligible. We fix $\sigma_1 = 0$ in the final fits.

Results

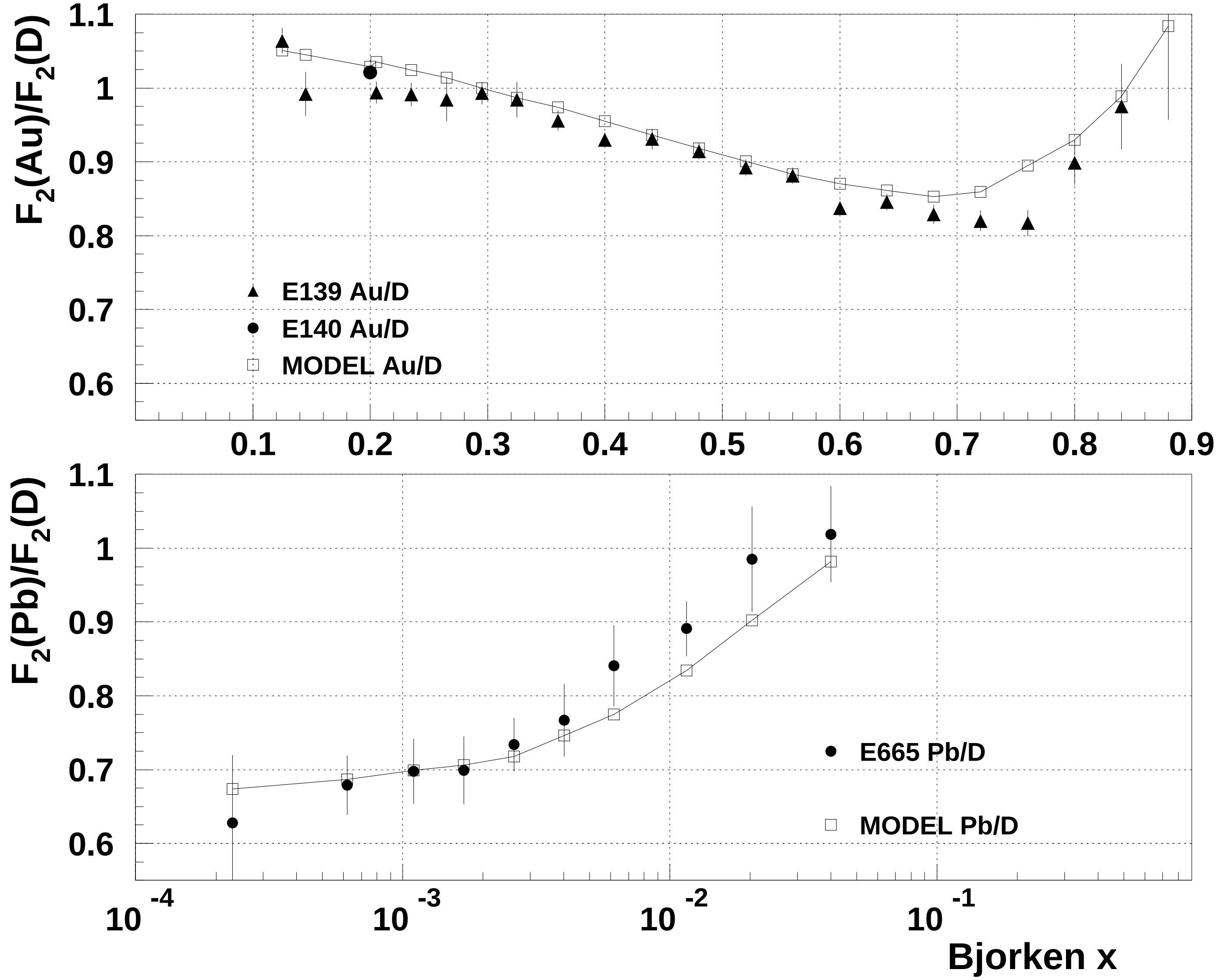
The model leads to a very good agreement with data on nuclear EMC effect. The x, Q^2 and A dependencies of the EMC ratios are reproduced for all studied nuclei (^{4}He to ^{208}Pb) in a 4-parameter fit with χ^2/d.o.f. $= 459/556$. For detailed discussion and comparison with data see S.K. & R.P., Nucl Phys A765(2006)126.

^{4}He/D



1.1
1
0.9
0.8
0.7
0.6
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
NMC He/D
E139 He/D
MODEL He/D
F2(He)/F2(D)
10 -4 10 -3 10 -2 10 -1
Bjorken x

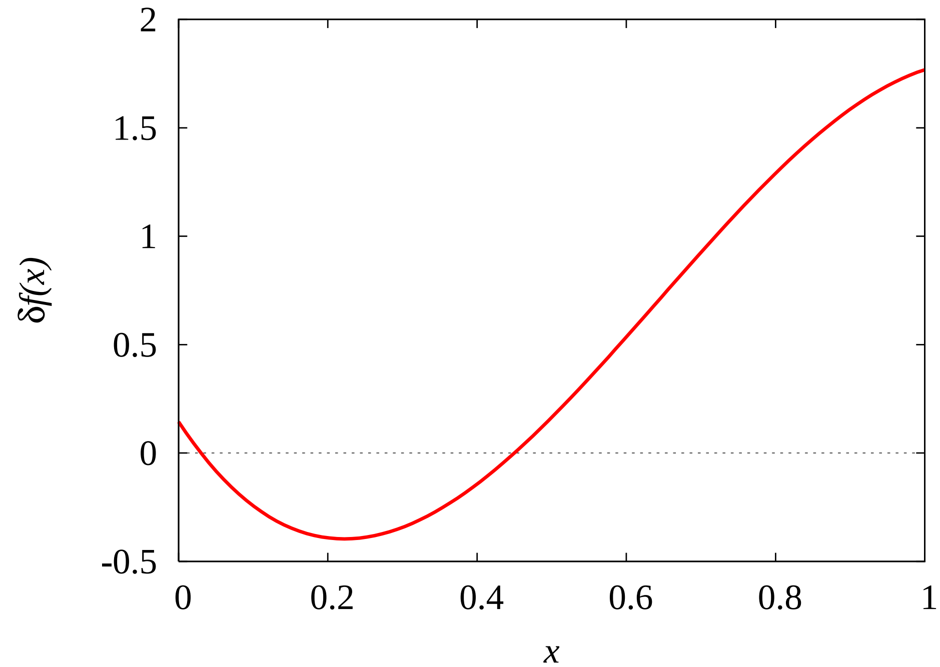
^{197}Au/D & ^{207}Pb/D



Off-shell function

The function $\delta f(x)$ provides a measure of modification of quark distributions in bound nucleon.

$$\begin{aligned} \delta f(x) &= C_N(x-x_1)(x-x_0)(1+x_0-x) \\ C_N &= 8.1 \pm 0.3 \pm 0.5 \\ x_0 &= 0.448 \pm 0.005 \pm 0.007 \\ x_1 &= 0.05 \end{aligned}$$



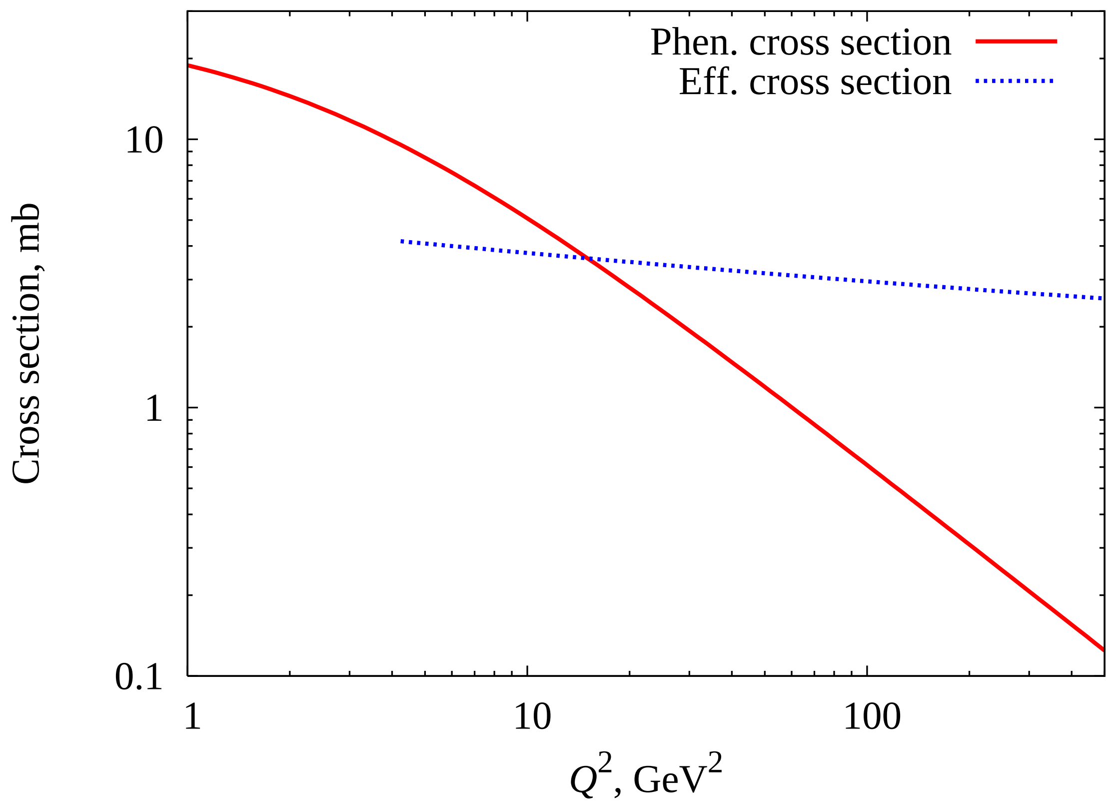
- Parameters from the global fit (all nuclei) are consistent with independent fits to different subsets of nuclei

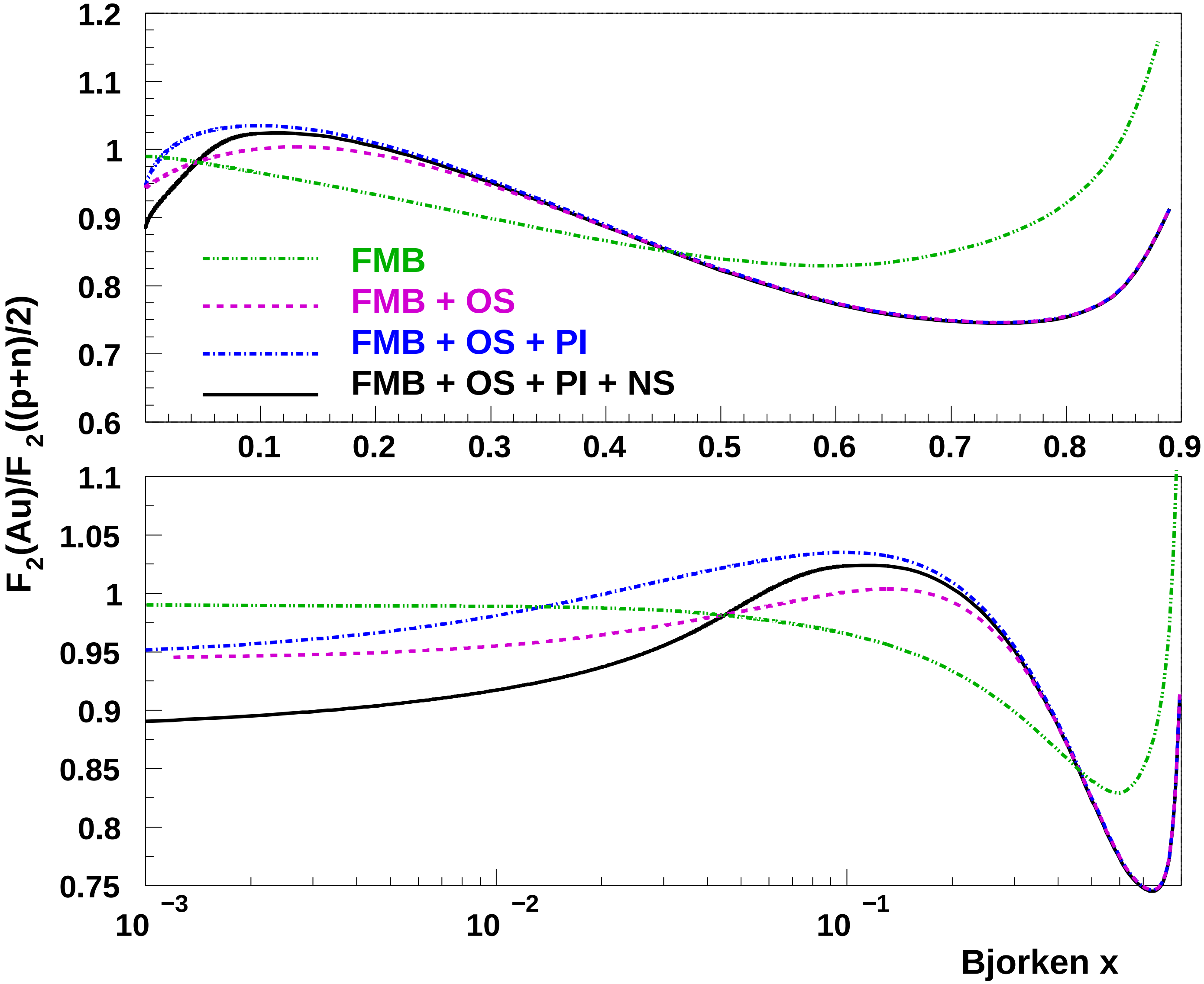
- The off-shell effect results in the enhancement of the structure function for $x_1 < x < x_0$ and depletion for $x < x_1$ and $x > x_0$.

Effective cross section

- The monopole form $\sigma_T = \sigma_0/(1 + Q^2/Q_0^2)$ with $\sigma_0 = 27\,\text{mb}$ and $Q_0^2 = 1.43 \pm 0.06 \pm 0.195\,\text{GeV}^2$ provides a good fit to existing DIS data on nuclear shadowing for $Q^2 < 20\ \text{GeV}^2$.

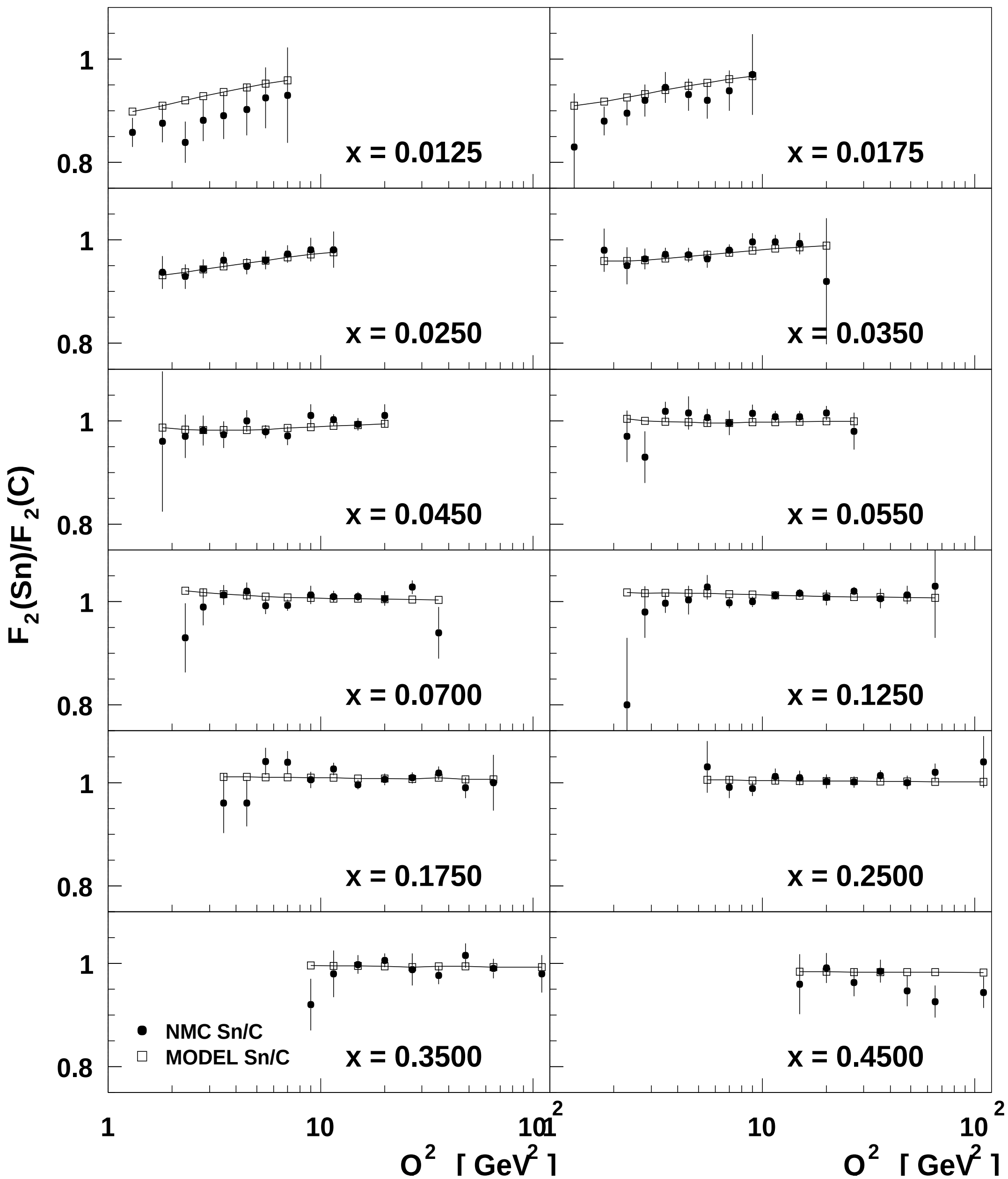
- Cross section at high Q^2 is not constrained by data. However, it can be calculated if we know δf. To do so we apply the model to nuclear valence quark distribution and require exact cancellation between off-shell (OS) and shadowing (NS) contributions to normalization: $\delta N_{\text{val}}^{\text{OS}} + \delta N_{\text{val}}^{\text{NS}} = 0$.

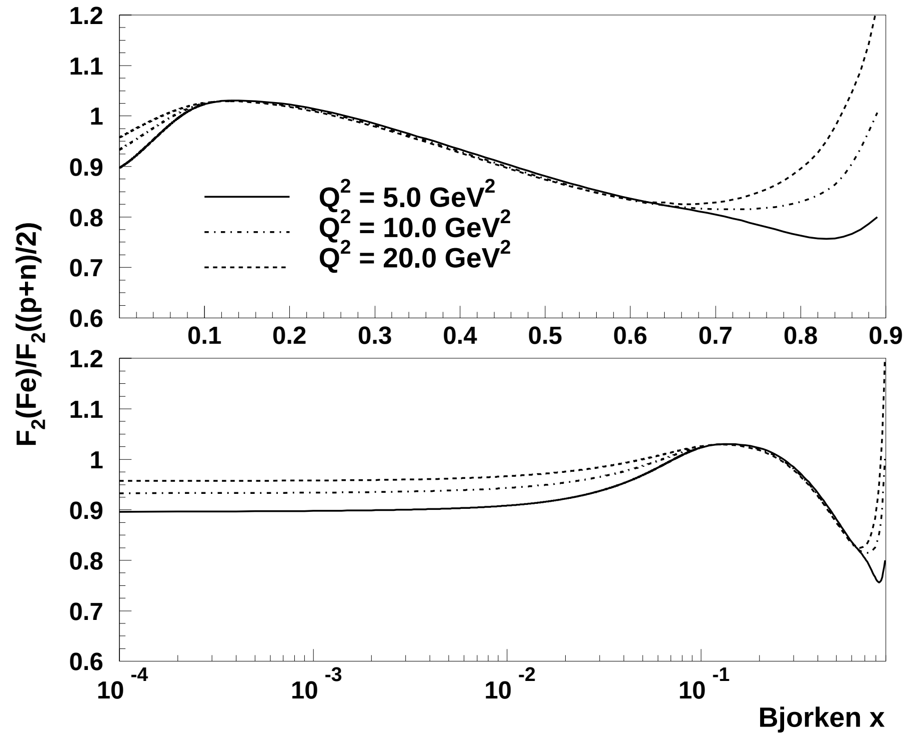




Different nuclear effects calculated for ^{197}Au at $Q^2 = 10\,\mathrm{GeV}^2$.

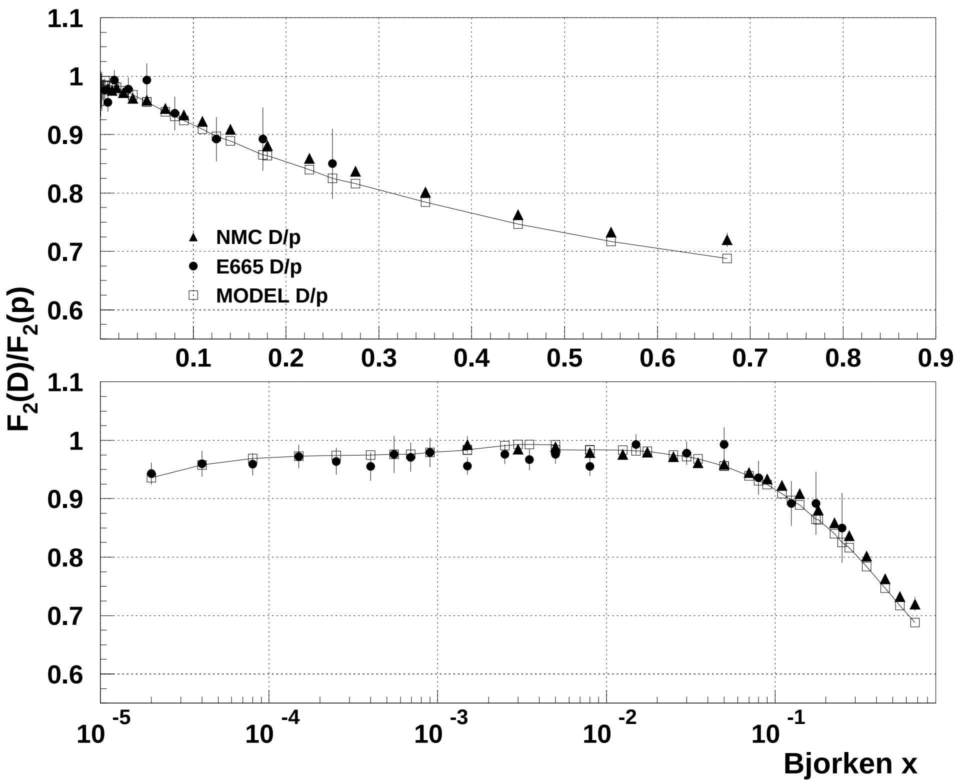
Q^2 dependence of $\mathcal{R}_2$



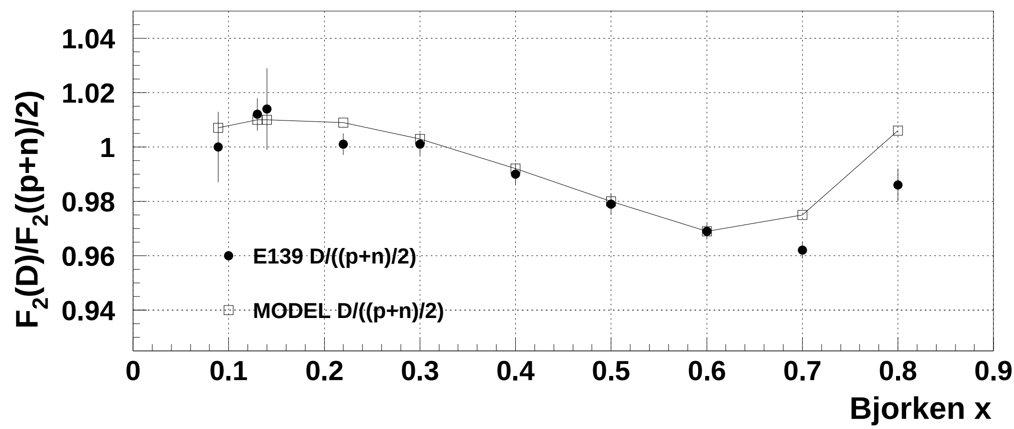


Q^2 dependence of $\mathcal{R}_2$ was observed for $x < 0.05$ (due to Q^2 dependence of shadowing effect) and for $x > 0.7$ (due to Q^2 dependence of target mass correction)

Deuteron structure functions



Comparison of E665 and NMC D$/p$ data to our calculations (curve with open squares). Note that these data were not used in our fit. The data points with $x < 10^{-3}$ also have $Q^2 < 0.5\ \mathrm{GeV}^2$.



Comparison of Gomez et.al. extraction of D$/(p+n)$ ratio from E139 nuclear data which was based on extrapolation and the nuclear density model of Frankfurt & Strikman (closed circles) to our calculations (curve with open squares).

Nuclear effects in neutrino DIS

- We apply the model developed for CL nuclear scattering for neutrino-nuclear interactions (for more details see S.K. & R.Petti, PRD46(2007)094023).

- Additional input is required to treat nuclear effects for νA scattering.

⇒ Treatment of axial current contribution at low x and low Q^2 is different from that of the vector current (PCAC). Relevant for F_L.

⇒ Off-shell corrections for different structure functions (F_2 and F_3) and its dependence on ν and $\bar{\nu}$.

⇒ Calculation of nuclear shadowing for $F_2^{\nu,\bar{\nu}}$ and $F_3^{\nu,\bar{\nu}}$ requires the amplitudes $a^{(I,C)}$ for different C-parity and isospin I. (the latter is important for accurate evaluation of isovector contributions, the neutron excess correction).

- DIS sum rules for nuclei (the Adler sum rule in the isovector channel and the GLS sum rule in the isoscalar channel) help to fix unknown amplitudes $a^{(0,-)}$ and $a^{(1,-)}$ responsible for (anti)shadowing corrections for $xF_3^{\nu+\bar{\nu}}$ and $F_2^{\bar{\nu}-\nu}$ combinations.

Neutrino cross sections

$$\frac{\mathrm{d}^2\sigma_{\mathrm{CC}}^{(\nu,\bar\nu)}}{\mathrm{d}x\mathrm{d}y} = \frac{G_F^2 ME/\pi}{(1+Q^2/M_W^2)^2}\sum_{i=1}^{5} Y_i F_i^{(\nu,\bar\nu)}$$

M and M_W are the nucleon and the W-boson mass, Y_i the kinematical factors, F_i the dimensionless structure functions.

$$Y_1 = y^2 x \frac{Q'^2}{Q^2}\left(1-\frac{m'^2}{2Q^2}\right),$$

$$Y_2 = \left(1-\frac{yQ'^2}{2Q^2}\right)^2 - \frac{y^2Q'^2}{4Q^2}\left(1+\frac{4M^2x^2}{Q^2}\right),$$

$$Y_3 = \pm xy\left(1-\frac{yQ'^2}{2Q^2}\right),$$

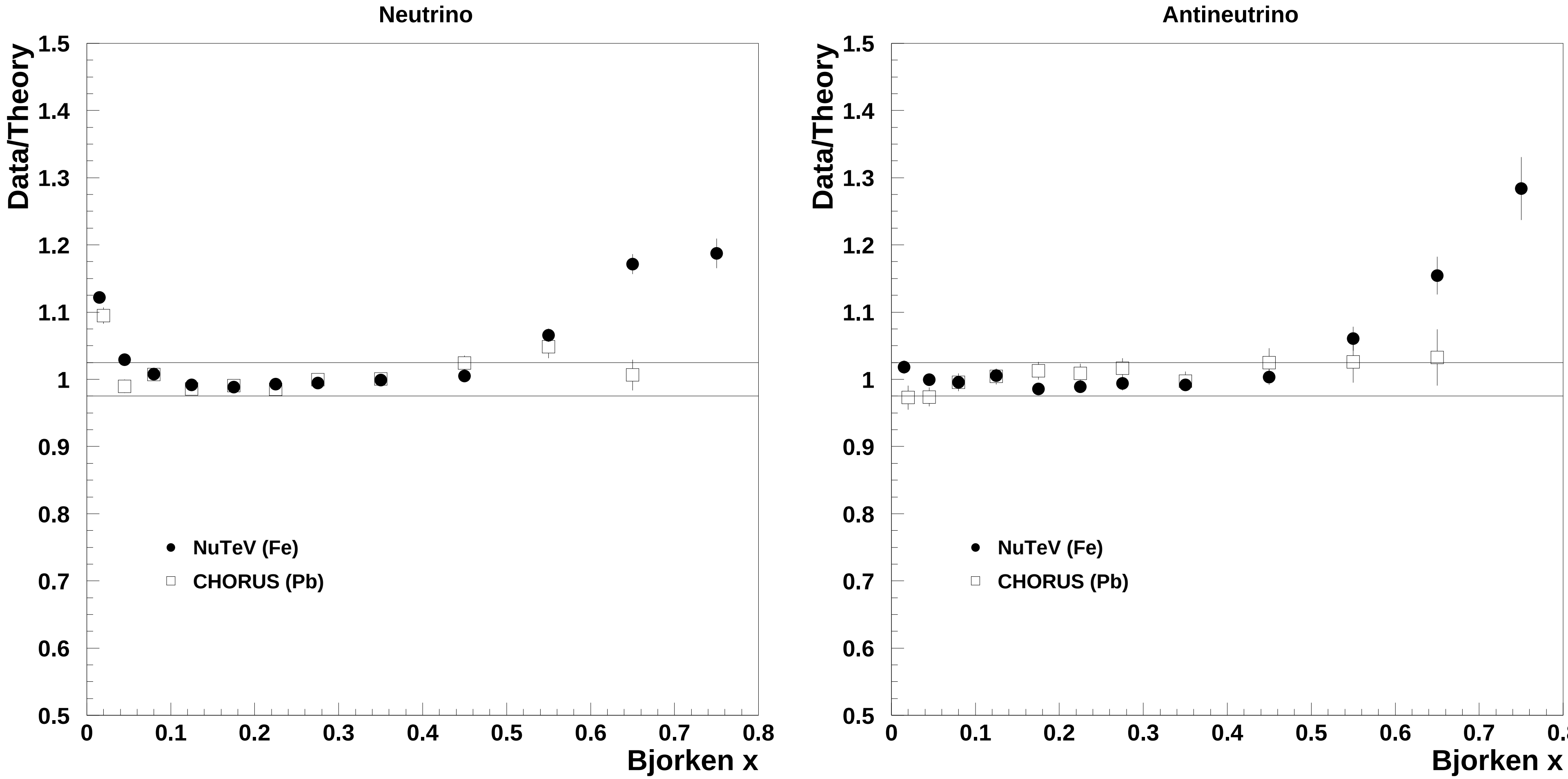
$$Y_4 = \frac{yQ'^2}{4Q^2}\frac{m'^2}{ME}, \quad Y_5 = -\frac{m'^2}{ME},$$

m' is the mass of the outgoing charged lepton and $Q'^2 = Q^2 + m'^2$, the sign $+(-)$ refers to neutrino (antineutrino) scattering.

Recently published cross section data:

Experiment	Beam	Target	Statistics	E values	x values	y values	# of points
NuTeV	ν	^{56}Fe	860k	35÷340	0.015÷0.75	0.05÷0.95	1423
	$\bar{\nu}$	^{56}Fe	240k	35÷340	0.015÷0.75	0.05÷0.85	1195
CHORUS	ν	^{208}Pb	930k	25÷170	0.020÷0.65	0.10÷0.80	607
	$\bar{\nu}$	^{208}Pb	160k	25÷170	0.020÷0.65	0.10÷0.80	607
NOMAD	ν	^{12}C	750k	20÷200	0.015÷0.65	0.15÷0.85	563

Data/Theory pulls for cross sections



The ratio of the measured differential cross-section and our calculation vs. x for neutrino and antineutrino interactions. The x-point is the weighted average over available E and y. The solid horizontal lines indicate a $\pm 2.5\%$ band.

χ^2 analysis

Cut	No. of data points		χ^2/d.o.f.	
	Neutrino	Antineutrino	Neutrino	Antineutrino
NuTeV (Fe)				
No cut	1423	1195	1.36	1.10
$x > 0.015$	1324	1100	1.15	1.08
$x < 0.55$	738	671	1.16	1.02
$0.015 < x < 0.55$	686	620	0.97	1.01
CHORUS (Pb)				
No cut	607	607	0.68	0.84
$x > 0.02$	550	546	0.55	0.83
$x < 0.55$	506	507	0.74	0.83
$0.02 < x < 0.55$	449	447	0.60	0.83

Values of χ^2 obtained from comparison of NuTeV and CHORUS cross section data with our calculations (not a fit).

Summary

- A detailed quantitative study of nuclear EMC effect was performed in a wide kinematical region of x and Q^2 and for nuclei from ^{4}He to ^{207}Pb. A model was developed which takes into account the QCD treatment of the nucleon structure functions and addresses a number of nuclear effects including nuclear shadowing, Fermi motion and nuclear binding, nuclear pions and off-shell corrections to bound nucleon structure functions.

- The off-shell effect is described in terms of a universal (independent of nuclei) function $\delta f(x)$, which makes the sense of a response of the nucleon quark distribution to a (small) variation of its invariant mass. The phenomenology of this function allows to describe x, Q^2 and A dependence of nuclear EMC effect in CL scattering.

- From the data-to-data comparison, we observe $\sim 2\%$ offset of the E03103 central points against previous SLAC E139, NMC measurements. A common renormalization E03103*0.98 brings the data sets in a perfect agreement [for more detail see Roberto Petti talk tomorrow].

- The model calculations of (anti)neutrino inelastic differential cross sections agree well with data on all studied targets [^{12}C (NOMAD), ^{56}Fe (NuTeV), ^{207}Pb (CHORUS)] for intermediate region of x.

- NuTeV data show excess over theory at large $x > 0.5$ for both ν and $\bar{\nu}$. However, this is not supported by CHORUS(Pb) and NOMAD(C) and also preliminary NOMAD(Fe) data [Roberto Petti, private communication].

- Both, NuTeV and CHORUS data show some excess over theory at small x $(0.015 - 0.025)$ [also supported by preliminary NOMAD(Fe) data – Roberto Petti, private communication].